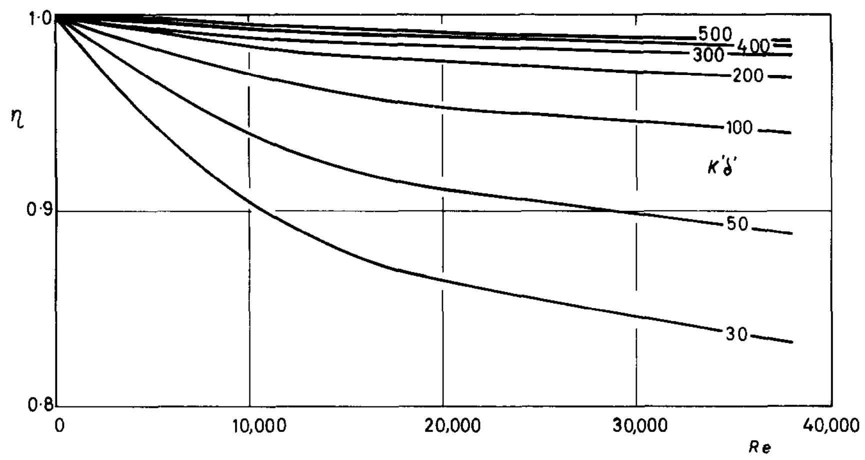

FIG. Effect of fin efficiency on heat exchanger performance.

These effects have been determined from the computer solution of the fin equation using the measured local heat-transfer coefficients; they are displayed by Fig. 6. The ordinate, η, is the ratio of the heat transferred by the real heat exchanger to that transferred by the ideal one.

5. CONCLUSIONS

The local heat-transfer data which have been obtained provide further material for testing recent theoretical procedures for predicting two-dimensional separated flows.

The greater constructional complexity of the finned-tube heat exchanger, relative to the conventional plain-tube heat exchanger, is not compensated by improved performance. The performance of the former is, in fact, significantly inferior to that of the latter. The fin-tube arrangement does not appear, therefore, to be a commercially attractive proposition.

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HEAT AND MASS TRANSFERS IN SOLID HELIUM

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NOMENCLATURE

b,	a constant;
t,	time;
x,	distance.

Greek symbol

α,	thermal diffusivity.

Abbreviations

b.c.c.,	body centred cubic;
h.c.p.,	hexagonal close packed;
L,	prefix used before a chemical symbol to denote the appropriate liquid.

INTRODUCTION

HELIUM forms a most interesting solid *at low densities* because the strong anharmonicity of its weak binding forces is very sensitive to changes of temperature, pressure and crystal perfection.

CRYSTAL GROWING

Cylindrical helium crystals (7 cm long, 5·5 mm dia.) were grown in stainless steel [1] specimen holders (0·25 mm thick). Carbon resistance thermometers were strapped and soldered to the outside of each specimen holder at 1·0, 3·5 and 6·0 cm from the effective ends of the heat sink and the heater-coil copper plug [2].

The technique of forming the crystals involved (i) cleaning the helium gas by pushing it, with an automatic Toepler pump, over activated charcoal contained in a U-tube immersed in LH_2; and (ii) subsequently maintaining, for at least twenty-five minutes, under constant pressure (by means of two pressure balances and a mercury transmitter), the specimen holder full of liquid helium at 0·05°K above the freezing temperature appropriate to the imposed pressure and helium isotope concentration [3–5]. The temperature of the heat sink, containing L^4He boiling at reduced pressure, filling capillary (of 0·3 mm bore) blocking where it passed through the heat sink, thereby isolating the specimens at constant volume. The crystals subsequently grew *downwards* as the contained liquid helium cooled and solidified.

^{4}He OBSERVATIONS

For the ^{4}He specimens which were cooled, at approximately one m°K min^{-1}, to the annealing temperature ($\sim$0·15°K below freezing), held there for periods in excess of 15 min, and then cooled at a rate of less than five m°K min^{-1}, the subsequent heat transfer measurements (see Fig. 1) were independent of time or particular increment of axial length of the crystal considered. Such data exhibit, over the Umklapp range, the exponential rise in conductivity with respect to the reciprocal of the absolute temperature as predicted by the quantum mechanical description of matter [6]. The high values of the conductivity compared with previously reported data [7–10] for similar densities indicate the good quality of the crystals so formed.

If rapid cooling of the specimen occurred without an enforced period of annealing then it is probable that imperfections—density gradients, vacancies, dislocations and

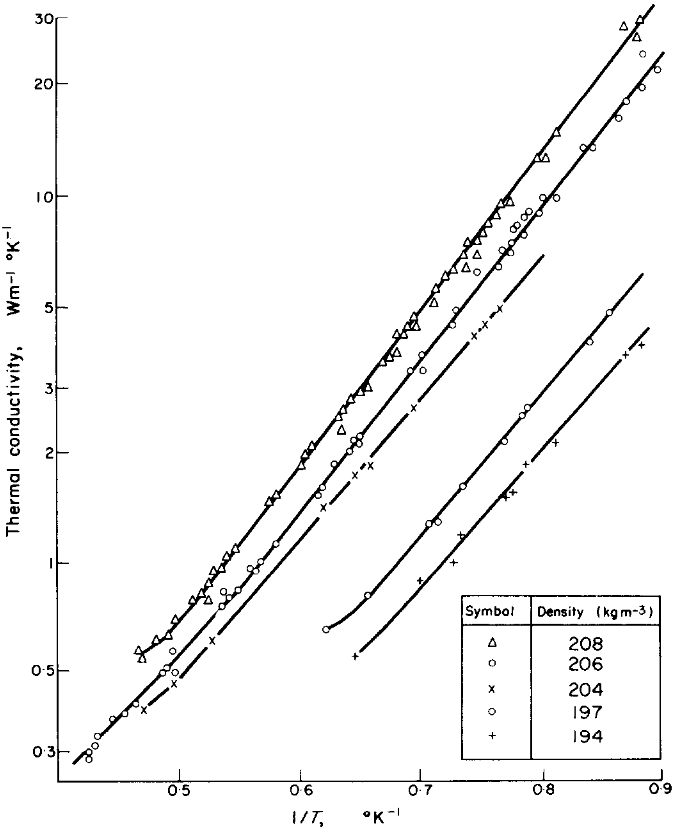

FIG. 1. The thermal conductivity of well-annealed ^{4}He

grain boundaries—were frozen-in the solid [10–12]. (Rogers deduced from thermal conductivity measurements on the b.c.c. phase of solid ^{3}He that there existed a high concentration of point defects of zero mass, such as vacancies, in his specimens but the data are still not fully understood [13]). The rate of self-annealing increases with temperature [14]. Self-annealing leads to apparent thermal conductivities which increase with time: during the present tests the apparent conductivity at 1·95°K of a 208 kg m^{-3} malformed ^{4}He specimen rose by 11 per cent in 284 min. The lower regions of such malformed crystals invariably exhibited lower conductivities (by $\lesssim 30$ per cent) than the upper parts due to the latter having been formed at a somewhat higher pressure.

^{3}He–^{4}He OBSERVATIONS

The dependence of the behaviour on the manner in which the specimens were grown was even more dramatic for solid ^{3}He–^{4}He mixtures. If a rapid cooldown from initial freezing to the annealing temperature occurred, then observations in qualitative agreement with those reported elsewhere [15–18] were obtained. However if very slow rates of cooling ($<$ one m°K min^{-1}) from initial to complete freezing of the helium were used then, due to the heat flush effect in the liquid, the ^{3}He tended to accumulate at the lower end of the specimen. However, complete phase separation of the ^{3}He and ^{4}He isotopes occurs in the solid only below 0·38°K [19], [20]. Quantitative measurements for a slowly formed crystal are presented in Fig. 2, which shows that the upper and lower regions of the specimen behaved differently with respect to time: the cross-over of the lines indicates the presence of another mechanism besides self-annealing. Such behaviour could be explained by the high-mobility ^{3}He isotopes diffusing *up* the crystal as predicted [22] by Mendelssohn.

For four very well-formed specimens, with mean ^{3}He in ^{3}He–^{4}He mixture concentrations of between 15 and 23 atomic per cent, a Dufour effect [23] was detected. This implies, for the present specimens, that associated with each rate of flow of the ^{3}He atoms relative to the ^{4}He there existed a temperature gradient. The introduction of a heat flux to the remote (i.e., lower) end of the specimen resulted in its temperature *falling*, though the mean temperature of the crystal, as expected, rose. This surprising effect amounted to a gradient of 12 m°K cm^{-1}, in the reverse direction to that anticipated, for a 16·9 atomic per cent ^{3}He in ^{3}He–^{4}He, 197 kg m^{-3} specimen. The magnitude of such temperature gradients, though only slightly influenced by the applied heat flux, were dictated by the crystal growing period during which the specimen holder contained both solid and liquid helium simultaneously. Eventually, as a result of the diffusion of the ^{3}He in the solid, positive values for the apparent thermal conductivity were obtained. Somewhat analogous behaviour has been reported previously [24, 25]. The cooling of the "heated" end of the helium mixture specimen was invariably accompanied by the observed crystal temperatures fluctuating with an amplitude of approximately two m°K and period 1·25 sec. This suggests that enantiotropic packing transitions were occurring. At the "low" pressures selected for this work, solid ^{4}He is an h.c.p. structure whereas solid ^{3}He has a b.c.c. lattice [4]: but it is doubtful if there is sufficient difference between the internal energies of the two structures [5, 26–28] to account for the present anomaly *completely*. A particular solution [2] of the second order differential equation describing the energy and mass diffusion in a solid which is absorbing energy, indicates that the temperature at a distance x along the crystal is dependent upon the exponential of $(\alpha b^2 t + bx)$ where t represents time, α the thermal diffusivity, and b a negative and possibly a complex number. The latter choice would lead to an effect in accordance with the observed oscillating temperatures.

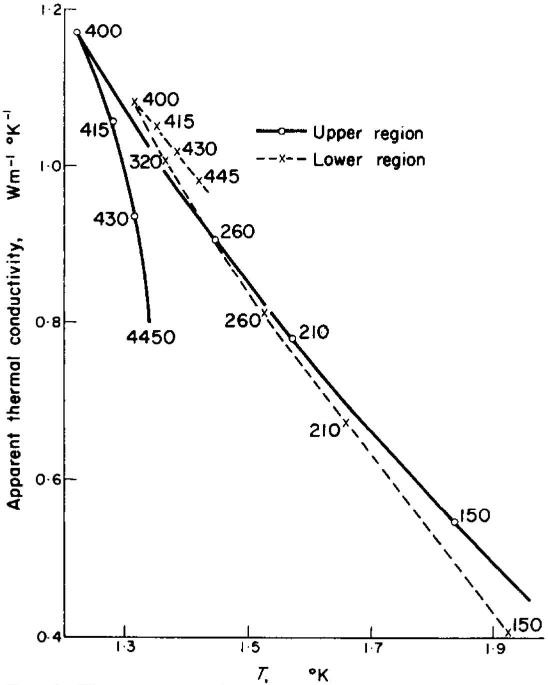

FIG. 2. The apparent thermal conductivities of the upper and lower regions of a vertical solid helium specimen of mean density 208 kg m^{-3} containing 6·7 atomic per cent by volume of ^{3}He in the ^{3}He–^{4}He mixture. The quoted number with each point indicates the period, in minutes, from the time of freezing of the upper end of the specimen.

CONCLUSIONS

Some of the disagreements [17, 29–31] between theoretical predictions and experimental observations for heat transfers through solid helium are to be expected because the crystals were imperfectly formed. The non-uniform distributions of defects and their subsequent migration through the crystals result in time dependent apparent thermal conductivities.

Further measurements are required on longer, more slowly grown, ^{3}He–^{4}He crystals formed in thinner specimen holders, so reducing their masking effect. To obtain even more dramatic observations, lower density ^{3}He–^{4}He crystals should be employed and attention given to the temperature regions below 0·38°K and near melting.

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